MATH 301 # TEST 4 (sample)

Spring 2017
Dr. Grzegorz Kubicki
<u>Work on it before the class meeting on April 06.</u>

1. (15 pts) Let R be a region bounded by the lines $y = x$, $y = x + 2$, $y = -x$, and $y = -x + 2$. Use the **change of variables** $x = u + v$, $y = u - v$ to evaluate the integral $\iint_R (x - y)\, e^{x+y}\, dy\, dx$.

2. (10 pts) Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 169$ that lies above the plane $z = 12$.

3. (12 pts) Use an unoriented line integral to evaluate **the mass of the wire** in the shape of the semicircle $x^2 + y^2 = 1$ ($y \geq 0$) with the mass density $f(x, y) = 2y$.

4. (13 pts) Use **spherical coordinates** to evaluate $\iiint_E z^2 \, dV$, where the solid region E lies in the first octant inside the sphere of radius 2 centered at the origin.

5. (20 pts)

(a) Justify that the vector field $\mathbf{F}(x, y) = (9x^2y^2 + x)\,\mathbf{i} + (6x^3y - y^2)\mathbf{j}$ is conservative.

(b) Find its potential $f(x, y)$.

(c) Use the Fundamental Theorem of Line Integrals to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any smooth curve in the plane between the points $(0, 0)$ and $(1, 6)$.

6. (10 pts) An object occupies the solid region is the first octant bounded by the coordinate planes, the planes $x = 2$, $y = 2$, and by the plane $z = 5 - y$. Its mass density at (x, y, z) is given by $f(x, y, z) = 1 + z^2$. Express its mass by a **triple integral.** Do not evaluate it.

7. (10 pts) Find the work done by the force **F** = -x **i** + y **j** over the semicircle
r = 2 cos t **i** + 2 sin t **j**, $0 \le t \le \pi$.

8. (15 pts) Use the **cylindrical coordinates** to prove that the **volume** of a cone with the base of radius R and height h is $V = (1/3)\, \pi\, h\, R^2$.