

MATH 301

TEST 2 (sample)

Spring 2017

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1. (10 pts) The function $f(x,y) = \frac{x^2 + y^2}{2x^2 - y^2}$ is undefined at $(0, 0)$. Does it have a **limit** as (x, y) approaches $(0, 0)$. ? Justify your answer.

2. (10 pts) Draw a **dependence diagram** for the composition of functions

$$w = f(x, y), \quad x = g(u, v), \quad y = h(t), \quad u = r(t), \quad v = s(t).$$

Use this diagram **to find a formula** for dw/dt .

3. (15 pts) A particle moves along the space curve $\mathbf{r}(t) = 3 \cos t \mathbf{i} - 3 \sin t \mathbf{j} + 5 \mathbf{k}$.

(a) Find the **velocity**, **acceleration**, **speed**, and **unit tangent vector** for this motion.

(b) Find the **distance** traveled by the particle during the time interval $[1, 3]$

(c) Find the **curvature** of the particle's path.

(d) Find **the normal and tangential components of acceleration** for this motion at $t = \pi$.

4. (10 pts) You can hit the golf ball such that its initial speed is 160 ft/sec. What should be **the angle of elevation** of the ball in order to clear a pond whose far edge is 120 yards away?

You might use a formula for the range $R = \frac{v_0^2 \sin 2\alpha}{g}$.

5. (15 pts) Let $f(x, y, z) = xy + 2xz + 4yz^2$.

(a) Find **the gradient** of f at $P(3, 2, 1)$ and the **directional derivative** of f at P in the direction of $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

(b) Find the **unit vector** in the direction of the most rapid increase of f at P and the **rate of increase** of f in that direction.

6. (15 pts) The volume of the cone of height h and with the circular base of radius r is $V = (1/3) \pi r^2 h$.

(a) Considering V as a function of two variables r and h find its **total differential** dV

(b) Find **the volume of the cone** with $r = 3$ and $h = 5$ (measured in inches)

(c) Use the differential from part (a) to **estimate the change in volume** of the cone if r increases from 3 to 3.02 inches and h decreases from 5 to 4.95 inches.

7. (15 pts) Find an equation of the **plane tangent** to the ellipsoid $(x-1)^2 + 4y^2 + z^2 = 9$ at the point $(3, 1, -1)$.

8. (15 pts) Find the **domain** of the function $f(x, y) = y/x$.

Sketch (and label) the **level curves** $f(x, y) = c$ for $c = 0$, $c = -1$ and $c = 2$.
Sketch the **gradient** of $f(x, y)$ at the point $P(1, 2)$ and at the point $Q(-2, 2)$.